Sequences and Series Using the TI-89 Calculator

- **Nonrecursively Defined Sequences**

A nonrecursively defined sequence is one in which the formula for the terms of the sequence is given explicitly. For example, \( a_n = \frac{n}{n+1} \) is the defining relation for a non-recursively defined sequence, while \( a_n = na_{n-1}, a_1 = 1 \) is an example of a recursively defined sequence.

Terms of a nonrecursively defined sequence can be generated using the built-in \texttt{seq} function of the TI-89. Select 3:List, 1:seq from the Math menu (Fig. 1), obtained by pressing the \textbf{Math} (= 2nd [5]) key.

To create the first 10 elements of the sequence defined by the formula \( a_n = \frac{n}{n+1} \), enter the expression shown in Fig. 3. This yields the sequence \( \{1/2, 2/3, 3/4, \ldots, 10/11\} \) as shown in Fig. 4.

To calculate \( \lim_{n \to \infty} \frac{n}{n+1} \) press the \textbf{Math} (= 2nd [5]) key and choose A:Calculus, 3:limit and enter the expression shown in Fig. 5. Infinity, \( \infty \), is the \( \bullet \) function of \textbf{CATALOG}. The limit of 1 is displayed on the home screen (Fig. 6).
Fig. 5 Syntax of the Limit Operator

\[ \lim_{n \to \infty} \frac{n}{n+1} \]

value at which to calculate the limit defining formula variable

Fig. 6 The Limit of a Sequence

\[ \lim_{n \to \infty} \frac{n}{n+1} \]

Plotting the Terms of a Sequence

The TI-89 has a built-in sequence plotting mode. To access it press the MODE key and set Graph to SEQUENCE (Fig. 7). The Y= screen changes to reflect the new mode (Fig. 8). Formulas for sequences are entered in the slots labeled u1, u2, etc. (The slots labeled u1, u2, etc. are only relevant for recursively defined sequences. See below.)

Fig. 7 Setting Graph Mode to Sequence

Fig. 8 Y= Window in Sequence Mode

I enter the sequence defined by the formula \[ a_n = \frac{n}{n+1} \] in u1. I select the number of terms to plot by entering the appropriate settings in the Window screen (press \[ \text{F2} \]). The nmin and nmax settings shown in Fig. 10 will generate the first 30 terms of the sequence. The xmin and xmax settings control how much of the sequence is displayed.

Fig. 9 Enter the Sequence Formula

Fig. 10 Window Settings

Sets the number of terms that are generated Controls what’s displayed
Fig. 11 shows the sequence plot. The numerical values of the terms of the sequence can be inspected using the Table features of the calculator. Press \textbf{TblSet} \((= \text{ \textbullet \ F4})\) to bring up the TblSet window (Fig. 12). The value of tblStart should agree with the Window variable \textit{nmin}. Press \textbf{TABLE} \((= \text{ \textbullet \ F5})\) to display the table (Fig. 13). Since sequences defined in the \textit{Y=} window are functions, they can be evaluated at the Home screen (Fig. 14).

- **Recursively Defined Sequences**

  Recursively defined sequences can be defined in the \textit{Y=} editor when Graph mode is set to sequence (Fig. 7). In Fig. 15 I define the sequence \(u_n = \frac{1}{3 - u_{n-1}}, \ u_1 = 2\). Note that the initial value is part of the definition of the sequence. Using the window settings in Fig. 16 I obtain a graph and a table (Figs. 17 and 18).
The TI-89 has many other sequence capabilities. For example, cobweb plots of recursively defined sequences can be constructed to study the long-term dynamics of a sequence. See the chapter of your manual entitled “Sequence Graphing” for more information.

**Infinite Series**

Partial sums of infinite series can be calculated on the Home screen by using the `seq` and `sum` functions together. (Both of these functions are found in the List submenu of the Math menu. See Figs. 1 and 2.) To find the sum of the first 10 terms of the sequence 

\[ a_n = \frac{n}{n+1}, \]

enter the expression 

\[ \text{sum(seq(n/(n+1),n,1,10,1))} \]

as shown in Fig. 19. If Mode is set to Auto or Exact, the answer can still be displayed in approximate mode by pressing \( \text{ENTER} \).

The sequence of partial sums of an infinite series can be constructed in the Data/Matrix Editor. Press the `APPS` key and choosing `6:Data/Matrix Editor` (Fig. 20) and then `3:New...` (Fig. 21).
Suppose we wish to find the partial sums of the series \( \sum_{i=1}^{n} \frac{1}{i^2} \). Move the cursor to the header cell \( c1 \) so that the cell is highlighted and press \( \text{ENTER} \). On the entry line use the seq function, obtained by pressing the \( \text{Math} \quad (= \text{2nd} \quad 5) \) key and then selecting 3:List, 1:seq, to create the formula to generate the sequence of terms of the series. In Fig. 22 I enter \( \text{seq}(1/n^2,n,1,10,1) \) to generate the first 20 terms of \( a_n = \frac{1}{n^2} \). Press \( \text{ENTER} \) to generate the sequence.

**Fig. 22** Generating a Sequence in the Data/Matrix Editor

Move to the header cell labeled \( c2 \) and press \( \text{ENTER} \) to access the entry line. Locate the \text{cumSum} function, obtained by pressing the \( \text{Math} \quad (= \text{2nd} \quad 5) \) key, and then selecting 3:List, 7:cumSum. Inside the parentheses, type \( c1 \) for the argument and then press \( \text{ENTER} \). \text{CumSum} is the cumulative sum function and it generates the partial sums of the series (Fig. 23). To see approximate values for the sums, change the Mode settings (Fig. 24) by pressing \( \text{MODE} \) and changing the Exact/Approx setting to Approximate. The data table will now display numerical approximations of the partial sums (Fig. 25).

**Fig. 24** Setting Approximate Mode

**Fig. 25** Approximate Values Displayed